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Application of Queuing Theory: Analysis of Services of Grocery Bazaar (GB) Supermarket, Akesan, Lasu Igando, Lagos State. Nigeria.

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Abstract

The formation of waiting line is a prevalence scenario that happens whenever the immediate demand for a service surpasses the current capacity to provide that service. This discrepancy may be temporal, but a queue accumulates during the period. Formation of a line causes an increase of customers waiting time, over utilization of the available servers and loss of customer's goodwill. Application of Queuing theory determines the measure of performance of the service facility which can be used to design the appropriate service facility. Data for this study was collected at Grocery Bazaar (GB) supermarket, Akesan Lasu Igando, Lagos State. This paper adopted the Single queue and Multi-channel models for the study of the existing structure of the Grocery Bazaar (GB) supermarket with three servers/machines. The data set was analyzed using R-studio software and the following estimates were obtained for Single Queuing Model $(M/M/1)$: Arrival Rate, Service Rate, Traffic Intensity, Average Time in the System, Average Time each Customer wait in the queue, Average Number of Customers waiting, Number of Customers in the System, Expected Average Total Time and Expected Number to be Served. Also, under the Multi-Channel Queuing System/Model $(M/M/C)$, this paper examined the Service Rate, Traffic Intensity, Probability of having zero Customer in the System, Average expected Queue Length, Average Number of Customers in the System, Expected Total Time, Waiting Time, Expected Number of Customers in the Queue, Probability of Queuing on the Arrival and Probability of not Queuing on the Arrival. The data collected was tested to show if it follows a Poisson and exponential distributions of arrival and service rate using chi square goodness of fit. The derived results revealed that the acquired data were statistically reliable to a great extent and suggestions were made at the end of the study on how to improve the process of queue /queuing problem in the Grocery Bazaar (GB) supermarket.

Keywords: Operations research**,** Queuing Theory, Arrival Process, Waiting Position, GB Supermarket.

1.0 Introduction

Operations research is concerned with the efficient allocation of scarce resources in both an art and science. The art lies in the ability to depict the concepts "Efficient and Scarce'' in a well-defined mathematical model of a given situation and the science consists in the deviation of computational methods for solving such models. The optimal allocation of money, manpower, energy or a host of other scarce factors is of importance to decision makers in many traditional disciplines. Therefore, Operations Research is concerned with the Scientific and Quantitative Techniques or approach used to solve managerial problems. Operational research (O.R) is the science of rational decision making and the study of design and integration of complex situation and system with the goals of predicting system behavior. The O.R. starts when mathematical and quantitative techniques are used to substantiate the decision being taken. Operations Research tools are not from any one discipline but it takes tools from different discipline such as Mathematics, Statistics, Economics, Psychology, Engineering, etc. and combines these tools to make a new set of knowledge for decision making. Today, O.R. became a professional discipline which deals with the application of scientific methods for making decision, and especially to the allocation of scarce resources. The main purpose of O.R. is to provide a rational basis for decisions making in the absence of complete information, because the systems composed of human, machine, and procedures may do not have complete information (Rahaman Aliyu, 2019).

Operations Research can also be treated as science in the sense it describing, understanding and predicting the systems behavior, especially man-machine system. Thus O.R. specialists are involved in three classical aspect of science, they are as follows:

(i) Determining the systems behavior
(ii) Analyzing the systems behavior by developing appropriate models and
(iii) Predict the future behavior using these models.

The emphasis on analysis of operations as a whole distinguishes the Operation Research from other research and engineering. Operations Research is an interdisciplinary discipline which provided solutions to problems of Military operations during World War II, and also successful in other operations. Today business applications are primarily concerned with O.R. analysis for the possible alternative actions. The business and industry befitted from O.R. in the areas of Inventory, Reorder Policies, Optimum Allocation and Size of Warehouses, Advertising Policies, etc. In Summary Operations Research is concerned with the optimum decision making or modeling all deterministic and probabilistic systems that originate from the real life. This applications occurring in Government, Business, Engineering, Economics, the Natural and Social Science are likely characterized by the needs to allocate limited resources.

1.1 History of Operations Research

Over the years there has been a remarkable development in the industries and business organization in the world. This development has however brought with its same problems needing serious attention. One problem is a tendency for the many component of an organization to grow into relatively hotonomous (independent) section with their own goals and value system which leads to losing sight of how their activities and objectives is embedded in those of the overall organization. A related problem is that as the complexity and specialization in an organization increase, it available resources to its various activities in a way that is most effective for the organization as a whole. This kind of organization as the need to find a better way to resolve them provided the environment for the emergent of operations research.

The roots of the Operations Research can be traced back by many decades, when early attempts were made to use a scientific approach in the management beginning of the activity called Operations Research has generally attributed been to the Military service early in the World war II. The war efforts brought an urgent need to allocate scarce resources to the various Military operations and to the activities within each operation in an effective manner (most effective way). Success of operations research in the Military led industries into gradually becoming interested in this new field. Early in 1936 the British Air Ministry established Bawdsey Research Station, on the East coast, near Felixstowe, Suffolk, as the Centre where all pre-war radar experiments for both the Air Force and the Army would be carried out. In 1937 the first of three major pre-war air-defense exercises was carried out in the summer of 1937. The experimental radar station at Bawdsey Research Station was brought into operation and the information derived from it was fed into the general air-defense warning and control system. In July 1938 a second major air-defense exercise was carried out. Four additional radar stations had been installed along the coast and it was hoped that Britain now had an aircraft location and control system greatly improved both in coverage and effectiveness. Accordingly, on the termination of the exercise, the Superintendent of Bawdsey Research Station, A.P. Rowe, announced that although the exercise had again demonstrated the technical feasibility of the radar system for detecting aircraft, its operational achievements still fell far short of requirements. He therefore proposed that a crash program of research into the operational - as opposed to the technical - aspects of the system should begin immediately. The term "Operational Research" [RESEARCH into (military) OPERATIONS] was coined as a suitable description of this new branch of applied science. The first team was selected from amongst the Scientists of the radar research group the same day.

In the summer of 1939 Britain held what was to be its last pre-war air defense exercise. It involved some 33,000 men, 1,300 aircraft, 110 antiaircraft guns, 700 searchlights, and 100 barrage balloons. This exercise showed a great improvement in the operation of the air defense warning and control system. On May 15th 1940, with German forces advancing rapidly in France, Stanmore Research Section was asked to analyses a French request for ten additional fighter squadrons (12 aircraft a squadron - so 120 aircraft in all) when losses were running at some three squadrons every two days (i.e. 36 aircraft every 2 days) In 1941 onward, an Operational Research

Section (O.R.S) was established in Coastal Command which was to carry out some of the most well-known O.R. work in World War II. The responsibility of Coastal Command was, to a large extent, the flying of long-range sorties by single aircraft with the object of sighting and attacking surfaced U-boats (German submarines). Thus the Operations Research started just before World War II in Britain with the establishment of teams of scientists to study the strategic and tactical problems involved in military operations. The objective was to find the most effective utilization of limited military resources by the use of Quantitative Techniques. Following the end of the war O.R. spread, although it spread in different ways in the U.K. and U.S.A.

In 1951 a committee on Operations Research formed by the National Research Council of USA, and the first book on "Methods of Operations Research", by Morse and Kimball, was published. In 1952 the Operations Research Society of America came into being. The responsibility of Coastal Command was, to a large extent, the flying of long-range sorties by single aircraft with the object of sighting and attacking surfaced U-boats (German submarines). The technology of the time meant that (unlike modern day submarines) surfacing was necessary to recharge batteries, vent the boat of fumes and recharge air tanks. Moreover, U-boats were much faster on the surface than underwater as well as being less easily detected by sonar. Thus the Operations Research started just before World War II in Britain with the establishment of teams of scientists to study the strategic and tactical problems involved in military operations. The objective was to find the most effective utilization of limited military resources by the use of Quantitative Techniques. Following the end of the war O.R. spread, although it spread in different ways in the U.K. and U.S.A. India was one the few first countries who started using Operations Research. In India, Regional Research Laboratory located at Hyderabad was the first Operations Research unit established during 1949. At the same time another unit was set up in Defense Science Laboratory to solve the Stores, Purchase and Planning Problems. In 1953, Operations Research unit was established in Indian Statistical Institute, Calcutta, with the objective of using Operations Research methods in National Planning and Survey. In 1955, Operations Research Society of India was formed, which is one of the first members of International Federation of Operations Research societies. Today Operations Research is a popular subject in management institutes and schools of mathematics.

1.2 Approaches in Solving O.R. Problems

Operations research has the following approaches in solving problems:

(1) Structuring the real life situation into a mathematical model, and bringing out the essential elements for the solution relevant to the objectives to be sort.

(2) Exploring the structures of such solution and developing systematic procedures for obtaining them

(3) Developing a solution including the mathematical theory, if necessary, that yields optimum values of the system measures of desirability.

1.3 Stages/Phases of Development of Operations Research

The stages of development of O.R. are also known as phases and process of O.R, which has seven important steps. These seven steps are arranged in the following order:

Step I: Observe the problem environment
Step II: Analyze and define the problem
Step III: Develop a model
Step IV: Select appropriate data input
Step V: Provide a solution and test its reasonableness
Step VI: Model validity
Step VII: Implement the solution

Step I: Observe the problem environment

The first step in the process of O.R. development is the problem environment observation. This step includes different activities; they are conferences, site visit, research, observations etc. These activities provide sufficient information to the O.R. specialists to formulate the problem.

Step II: Analyze and define the problem

This step is analyzing and defining the problem. In addition to the problem definition, the objectives, uses and limitations of O.R. study of the problem also defined. The outputs of this step are clear grasp of need for a solution and its nature understanding. In other words, this involves defining the scope of the problem under investigation with aim of identifying the three principal elements of decision problem; decision variables, objectives function and specification of the constraints.

Step III: Develop a model

This step develops a model; a model is a representation of some abstract or real situation. The models are basically mathematical models, which describes systems, processes in the form of equations, formula/relationships i.e. this entails the translation of the problem definition into mathematical relationships. The different activities in this step are variables definition, formulating equations etc. The model is tested in the field under different environmental constraints and modified in order to work. Sometimes the model is modified to satisfy the management with the results.

Step IV: Select appropriate data input

A model works appropriately when there is appropriate data input. Hence, selecting appropriate input data is important step in the O.R. development stage or process. The activities in this step include internal/external data analysis, fact analysis, and collection of opinions and use of computer data banks. The objective of this step is to provide sufficient data input to operate and test the model developed in Step III.

Step V: Provide a solution and test its reasonableness

This step is to get a solution with the help of model and input data. This solution is not implemented immediately, instead the solution is used to test the model and to find there is any limitations. Suppose if the solution is not reasonable or the behaviour of the model is not proper, the model is updated and modified at this stage. The output of this stage is the solution(s) that supports the current organizational objectives. This is achieved by using well-defined optimization techniques (e.g. linear, nonlinear, integer programming) to yield an optimal solution.

Step VI: Model Validity

This step entails checking whether or not the proposed model does what it is supposed to do. That is, does the model predict adequately the behaviour of the system under study?

Step VII: Implement the solution

At this step the solution obtained from the previous step (i.e. Step V) is implemented. The implementation of the solution involves many behavioral issues. Therefore, before implementation the implementation authority has to resolve the issues. A properly implemented solution results in quality of work and gains the support from the management. This involves the translation of the results into operating instructions issued in understandable form to the individual who will administer the recommended.

1.4 Model Building in Operations Research

Putting the summary given above into simpler form, the usual phases of an operations research study can be summaries thus:

i. Formulating the problem,
ii. Constructing a mathematical model to represent the system under study,
iii. Deriving a solution from the model,
iv. Testing the model and the solution derived from it,
v. Establishing controls over the solution and
vi. Putting the solution to work.

Hence, the first thing to do is formulating the problem, before the model is built from it. Most practical problems are initially communicated to an operations research team in a vague/imprecise way. Hence, there is the need to study the relevant system and develop a well define statement of the problem being considered.

After formulating the decision maker's problem, then the problem is transform into something convenient for analysis. The form convenient for analysis in Operations Research is a mathematical model that represents the essence of the problem. In everyday life so also the mathematical models are idealized representation, but they are expressed in terms of mathematical symbols and expressions.

1.5 Effects of Models in Operations Research

There are many advantages of mathematical model over a verbal description of the problem.

(1) Mathematical model describes a problem mare precisely. This makes the overall structure of the problem more comprehensible and it helps to re-view important cause-and-effect relationships.

(2) It indicates more clearly what additional data relevant to the analysis.

(3) It facilitates dealing with the problem in its entirety and considering all its interrelationships simultaneously.

(4) Mathematical model forms a bridge to the use of high-powered mathematical techniques and computers to analyze the problem. Many of the components of the model may entail the use of package software.

Operations Research uses any suitable tools or techniques available. The common frequently used tools/techniques are mathematical procedures, cost analysis, electronic computation. However, operations researchers given special importance to the development and the use of techniques like Linear Programming, Transportation Problem, Assignment Problem, Game Theory, Decision Theory, Queuing Theory, Inventory Models and Simulation.

1.6 Aim and Objectives

The aim of this research is to look into various ways for the cause of long queuing (waiting of customer and how to improve on the service). The Objectives of the Study are:

1. To determine the average number of customer in the queue and service.
2. To estimate the average waiting time a customer used in the queue and system.
3. To determine measures of effectiveness of the system and to design a queuing model that will optimize the services at Grocery Bazaar (GB) supermarket.
4. To determine the efficiency of the market and build queuing theory models for the markets at Grocery Bazaar (GB) supermarket.
5. To show if the data collected follows a Poisson and exponential distributions of arrival and service rate using chi square goodness of fit.

1.7 Coverage

The data cover the number of customers that arrived at the Grocery Bazaar (GB) supermarket, the arrival time, service duration, departure which is the time-out and the number of customers was recorded in the data. There are three attendants which serve the customers one after the other.

1.8 Scope and Limitation

Queuing theory is the aggregate of items awaiting service. It objectives functions is to know the situation where there is a flow of customers arising at one or more service facility.

i. There may be problem of shutting i.e. some people move from back to the front.
ii. Some people, after being on the queue they may decide to leave which definitely affect the record.
iii. The result is limited to the service of the supermarket.
iv. In a situation where the service seems to be partial i.e. some customers may be served at random on last come last serve

2.0 Methodology of the Study

The Single and Multi-Channel Queuing System will be adopted to the data set collected with a view to determine the best method to be used by the Grocery Bazaar (GB) supermarket which will determine the average number of customers in the station.

2.1 Research Design and Area of Study

The main aim of this research is to show how the management of Grocery Bazaar (GB) supermarket will go about the reduction of the waiting time of customers. This paper will also check if increasing the number of server will reduce the waiting time as well as putting the profit in consideration. Hence, the objectives of this paper will be achieved by analyzing the real life observe data, then constructing a new model of system and using statistical analytical tools like poisson, exponential and chi-square distribution to study the pattern and reaction to change in the system. This research paper center on the waiting area of the Grocery Bazaar (GB) supermarket.

2.2 Formulation of Model for The System

To formulate a model for this system, we put the following assumptions into consideration:

1. The arrival of customers into the system is discrete form poisson distribution with arrival rate λ.
2. The queuing discipline is first come first serve.
3. There is only three services channel i.e. the $(M/M/3)$.
4. The service channel can only render service of finite rate exponentially distributed with service rate μ.
5. The calling population (i.e. the number of customers calling for services is finite).
6. The number of customers in the system at the time of initiating the observation is assumed to have arrived in the first unit time.

7. The waiting area for the customers in the system is N, which is either limited or unlimited. Hence, the model can be formulated appropriately by using a system for the investigation system. Kendall's notation is introduced; $(V/W/X/Y/Z)$.

Where:

$V =$ The arrival distribution or pattern is poisson as indicated earlier.
$W =$ The service time distribution is exponential as indicated earlier.
$X =$ The number of available server in the system is two from the assumption above.
$Z =$ This represents the queue discipline which is First Come First Served (FCFS).

Hence, with the above assumptions and approach the formulated model is $(M/M/3/N/FCFS)$ by Kendall's notation.

2.3 Solution of The Defined Problem

There is no unique model for single channel; the particular approach one adopts depends on some factors. These factors include the purpose of the solution exercise, the information available about the system and the tools for the work of the system. For this investigation, analytical method of solution is adopted.

2.3.1 Algorithm for Solving the Problem

1. Considering the system characteristics
2. Collection of data based on:
   a. Arrival distribution
   b. Service time distribution
3. Estimating the parameter λ and μ from the data.
4. Testing of the data distribution for statistical conformity to the assumed theoretical probability distribution using the chi-square tests for goodness for fit.
5. Analytical solution to the system model using the values of the parameters λ, μ and N (system capacity) calculated from the data.

2.4 Method of Data Collection

The data was collected primarily by direct observation at Grocery Bazaar (GB) supermarket. Thus the researcher recorded the following events as it happened in the system using a wrist watch.

1. The time of arrival of each customer.
2. The time service commences for each customer in the system.
3. The departure time of the customer in the system.

These events were observed at three service point of the supermarket. A form was designed for this exercise and the above required information was recorded in the form.

2.5 Sources of Data

The data used in this study was collected at the Grocery Bazaar (GB) supermarket at Akesan Lasu Igando Lagos. It was collected in a day; in the data have the record of the arrival time, service time, duration (total time) departure time and the number of people on the queue, which is up to twenty. The data is a primary data.

3.0 Literature Review

Queuing theory is a mathematical study of waiting line or queue. The theory enables mathematical analysis of several related process, including the arrival at the (back of the) queue (essentially a storage process) and being served at the front of the queue. The queue permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number waiting or receiving service and the probability of encountering the system in certain states such as empty, full, having an available server or having to wait a certain time to be served. Queuing theory has become one of the most important, valuable and arguable one of the most universally used tools by an operational researcher. It has application in diverse fields including telecommunication, traffic engineering, computing and design of factories, shops, office, banks and hospitals.

Some scholars maintain that we queue or wait in line to get served in commercial outfits like checkout counters, banks, super markets, fast food restaurants etc. (Kavitha; Palaniammal, 2014), grocery stores, post offices, to waiting on hold for an operator to pick up telephone calls, waiting at an amusement park to go on the newest ride (Mandia, 2009). Others according to the authors include: waiting in lines at the movies, campus dining rooms, the registrar's office for class registration, at the Division of Motor Vehicles etc. We equally queue in other socio-political settings like queuing to vote, waiting in line to be attended to by a public servant in government offices, etc. Waiting causes not only inconvenience, but also frustrate people's daily lives. Thus, unmanaged queues are detrimental to the gainful operation of service systems and results in a lot of other managerial problems (Yakubu; Najim, 2011).

(Seigha Gumus; Gordon M.B; Mobolaji H.O., 2017) providing insight into the study of queue theory through the examination of the Blue Meadows Waiting Line Model, our work presents a foundation for the development of strategies that may enhance customer satisfaction in fast food restaurants and other service industries. We evaluated the performance of single channels, two servers in Blue-Meadows restaurant at the University of Benin. (Lucey, 2015) define queue as aggregate number of items or customer awaiting service function. In other words, any group of people or object waits their turn to be served, attended to configure a queue and gave queuing theory as a construction of quantitative model of various type of queuing system so that prediction may be made about how the system will cope with demand made upon it. (Gautam Chaudhary and Ke J. C., 2014) incorporated an unreliable retrial queue with delaying repair and general retrial times under Bernoulli vacation schedule. Recently queuing process and its application to customer service delivery was proposed by (Bakari, H.R, Chamalwa, H.A and Baba, A.M, 2014). (Ying – Lin Hsu, Ke J.C, Tzu -Hsin Liu and Chia -huang, Wu, 2014) gave modelling of multi-server repair problem with switching failure and reboot delay under related profit analysis. (Liu Hsin Tzu and Ke J.C., 2014) has studied on the multi-server machine interference with modified Bernoulli vacations. (Yue D., Yue, W and Qi, H., 2013) performed performance analysis and optimization of a machine repair problem with warm spares and two heterogeneous repairmen.

A Queuing model of a system is an abstract representation whose purpose is to isolate those factors that relate to the system ability to meet service demands whose occurrences and durations are random (Sztrik, 2010). The study of the queue deals with quantifying the phenomenon of waiting in lines using representative's measures performances, such as average queue length, average waiting time in queue and average facility utilization (Taha, 1976). Queuing models provide the analyst with a power tool for designing and evaluating the performance of queuing systems. (Bank, et al, 2001). Any system in which arrivals place demand upon a finite capacity resource may be termed as queuing system, if the arrival times of these demands are unpredictable or if the size of these demands are unpredictable, then conflicts for the use of the resource will arise and queue of waiting customers will form and the length of these queue depend on two aspects of the flow pattern: first, depends on the average rate, secondly, they depend on the statistical fluctuation of this rate (Lenrock, M., 1975).

In 1990, the first study of queuing theory was done by Danish mathematician, Erlang which resulted into the worldwide acclaimed, Erlang telephone model. He examined the calls on utilization of automatic dial equipment. The original problem Erlang treated was the calculation of this delay for one telephone operator and in 1917, the results were extended to the activities of several telephone operators. That was the same year that Erlang published his well-known work" solution of some problem in the theory of probabilities of significance in the field of telephone traffic continued largely along the lines initiated by Erlang and the main publication were those of (Molina, 1927; Thornton D. Fry Kin 1928). (Lucy, 2001) gives definition of a queue for queuing purpose as aggregate number of items or customers awaiting a service function. In other words, any group of people or objects wait their turn to be served attended to constitute a queue theory as the construction of qualitative model of various type of queuing system so that prediction may be made about how the system will cope with demand made upon it.

4.0 Queuing Theory

Queuing theory is a branch of Mathematics that studies and models the acts of waiting in lines. As we know queues are a common every-day experience. Queues form because resources are limited. In fact, it makes economic sense to have queues. For example, how many supermarket tills you would need to avoid queuing? How many buses or trains would be needed if queues were to be avoided or eliminated? So, in designing queuing systems we need to aim for balance between service to customers (short queues implying many servers) and economic considerations (not too many servers). This chapter will take a look into the formulation of queuing theory along with examples of the models and applications of their use. The goal of this section is to provide the readers with enough background in order to properly model a basic queuing system into various categories. Also, the readers should begin to understand the basic ideas of how to determine useful information such as average waiting times from a particular queuing system. (Richard et al., 2004) define Queuing system as birth-death process with a population consisting of customers either waiting for service or currently in services. Hence, a birth occurs when a customer arrives at a service facility, a death occurs when a customer departs from the facility.

The first research on quelling theory, "The Theory of Probabilities and Telephone Conversations" was published in 1909 by A.K. Erlang, now considered the father of the field. His work with the Copenhagen Telephone

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Company is what prompted his initial foray into the field. He pondered the problem of determining how many telephone circuits were necessary to provide phone service that would prevent customers from waiting too long for an available circuit. In developing a solution to this problem, he began to realize that the problem of minimizing waiting time was applicable to many fields and began developing the theory further.

Queuing theory is the mathematics of waiting lines. It is extremely useful in predicting and evaluating system performance. Also, it has been used for operations research and then, traditional queuing theory problem refers to customers visiting a store, filling station, analogous to request arriving at a device. In other words, queuing theory is the mathematical study of waiting lines or queues. A queuing model is constructed so that queues lengths and waiting time can be predicted. It examines every component of waiting in lines to be served, including the arrival process, service process, number of servers, number of system place and the number of "customers" (which might be people, data pockets, cars etc.). Typical examples might be:

- Banks/Supermarkets – waiting for service;
- Computers – waiting for a response;
- Failure situations – waiting for a failure to occur e.g. in a piece of machinery;
- Public transport – waiting for train or a bus.

As we can see from observation of people in life, which involved in queuing, which takes on important part? The observation can be carry-out in some areas like in hospitals, patients waiting to see doctor, customer in the supermarket, cashier desk, petrol filling station, and talking transport from one place to another. In all the examples given, there are two things common to them all which is the arrival of people and items requiring service daily (i.e. waiting time) when the service is busy.

The question arises here is how one can find a balance between cost associated with having to wait for service and the costs of providing a fast service? The set of tools and techniques for analysis such problems, concerned with providing service to customers in a line is called **Queuing Theory or Waiting Line Models.**

4.1 Queuing Characteristics

The following are the queuing characteristics:

(i) Arrival process
(ii) Behaviour of customers
(iii) Service time distribution
(iv) Number of servers
(v) System capacity or service capacity
(vi) Population size
(vii) Service discipline
(viii) Waiting room

Each of these is described mathematically as follows:

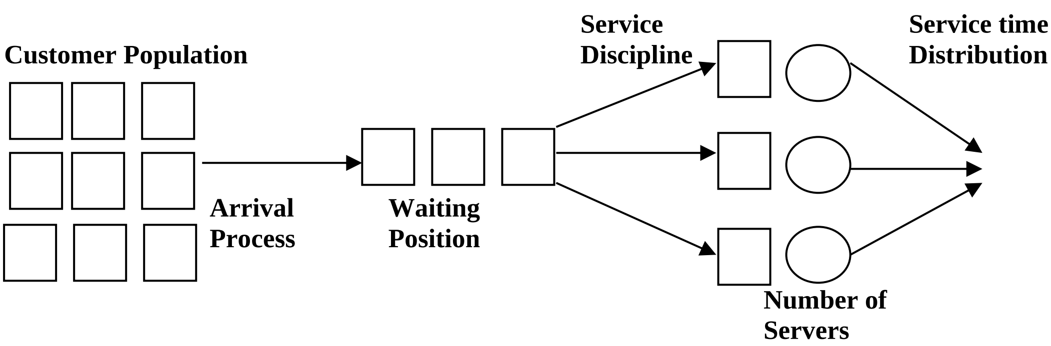

Figure 4.1: Queuing Characteristics

(i) **The Arrival Process of Customers:** Usually we assume that the inter-arrival times are independent and have a common distribution. In many practical situations, customers arrive according to a poisson stream (i.e. exponential inter-arrival times), customers may arrive one by one, or in batches. An example of batch arrivals is the customs office at the border where travel documents of bus passengers have to be checked.

(ii) **The Behaviour of Customers:** Customers may be patients and willing to wait (for a long time) or customers may be impatient and leave after a while. For example, in call centers, customers will hang up when they have to wait too long before an operator is available, and they possibly try again after a while.

(iii) **The Service Time Distribution:** Usually we assume that the service time are Independence and Identically Distributed (IID) and that they are independent of the inter-arrival time. The service time can be deterministic or exponentially distributed. It can also occur that service times are dependent of the queue length. For example, the processing rate of the machines in a production system can be increased once the number of jobs waiting to be processed becomes too large.

(iv) **The Number of Servers:** Servers may or may not be identical. Service discipline determines allocation of customers to servers.

(v) **The Service/System Capacity:** There may be a single server or a group of servers helping the customers. Also, maximum number of customers in the system (including those in service) may be finite or infinite.

(vi) **The Population Size:** The total number of potential customers may be finite or infinite.

(vii) **The Service Discipline:** Customers can be served one by one or in batches. We have many possibilities for the order in which they enter service. We mention:
   (a) First-Come-First-Serve (FCFS) or First-In-First-Out (FIFO) i.e. in order of arrival which means, who comes earlier, leaves earlier. FIFO is the most common service discipline.
   (b) Last-Come-First-Out (LCFO) or Last-In-First-Out (LIFO) i.e. who comes later leave, earlier. E.g. in a computer stack or a shunt buffer in a production time.
   (c) Service-In-Random-Order (SIRO) or Random-Service (RS) i.e. the customer is selected randomly.
   (d) Priorities: E.g. rush orders first, shortest processing time first
   (e) Processor Sharing (PS): In computer that equally divide their processing power over all jobs in the system.

(viii) **The Waiting Room:** There can be limitations with respect to the number of customers in the system. For example, in a data communication network, only finitely many cells can be buffered in a switch. The determination of good buffer sizes is an important issue in the design of these networks. Therefore, the particular discipline chosen will likely greatly affect waiting times for particular customers (nobody wants to arrive early at an LCFO discipline). As expected, service discipline affects the nature of the stochastic process that represents the behaviour of the queuing system.

4.2 Assumptions of Queuing Theory

The following are basic assumptions underlying common queuing models:

(i) Arrivals are served on a FIFO basis i.e. Independent arrivals.
(ii) Every arrival waits to be served regardless of the length of the line (queue) i.e. there is no balking or reneging.
(iii) Arrivals are independent of preceding arrivals (the arrival rate) does not change over time.
(iv) Arrivals are described by a poisson probability distribution.
(v) Large queue does not discourage customer.
(vi) The inter-arrival time has an exponential probability distribution with a mean arrival rate of one customer arrivals per unit time.
(vii) The source population has infinite size.
(viii) There is no unusual customer behaviour.
(ix) The service discipline is FIFO or FCFS
(x) The service time has an exponential probability distribution with a mean service rate of$m$ service completions per unit time.
(xi) The mean arrival rate (λ) is less than the mean service rate (μ) i.e. $\lambda < \mu$.
(xii) There is no unusual server behaviour.

4.3 Importance of Queuing Theory

(i) In queuing theory, a model is constructed so that queue lengths and waiting times can be predicted.
(ii) Queuing theory is generally considered a branch of operations research because the results are often used when making business decision about the resources needed to provide a service.

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(iii) A queuing model could be used to study the lifespan of the bulbs in street lamps in order to better understand how frequently they need to be replaced.

(iv) Tracking: Companies use several forms of queues to address different situations and goals. Help desk and customer service departments often create virtual queues, assigning people needing service case number and priority statuses. These help technicians and specialists stay on top of all the situations and cases before them. For example, a company's. IT help desk may serve hundreds and even thousands of employees using personal computers, mobile devices and proprietary devices. This requires a detailed and comprehensive tracking system to help manager efficiently allocate their team member's time and expertise.

(v) Timeliness: Business conduct studies using mathematical models formulas to determine the best way of serving the greatest number of customers, given their staffing resources. In retail businesses, the volume of transactions is extremely important in maximizing revenues and profitability which means lines and queue are critical. Factors, including the number of workers, customers' volume and available equipment, go into the calculations that result in queue formats. These account for why supermarkets typically operate multiple lines/queues using several check stands, while banking and airlines usually use long queues that lead to several or even many tellers and agents.

(vi) Insufficiencies: Companies must stay on top of their queuing practices to achieve the best result. An organization, such as the bank, needs to stock to its model once it's been determined that maximum efficiency can be achieved, both in labour costs and customers served, by using a centralized queue based on staffing at least three tellers during peak hours. The loss of one teller may cost the bank significantly in lost opportunities-transaction that don't occur because customers don't want to wait in line (queue) or even accounts closed by customers who feel they can get better treatment at another bank. Conversely, adding too many extra tellers could inordinately raise labour costs.

(vii) Assessments: From time to time, companies should reassess their queuing strategies to ensure they remain effective. As business goals and operations shift, a queuing system may become less useful or efficient than it once was. For instance, a bank branch that reduced its staff to only two tellers could more efficiently serve customers using two lines (queues), rather than one. Similarly, a customer service call Centre may improve customer satisfaction and achieve more by returning customer calls, rather than mandate long caller hold times.

(viii) Queuing theory can be applicable in many real world situations. For example, understanding how to model a multiple server queue could make it possible to determine how many servers are actually needed and at what wage in order to maximize financial efficiency.

(ix) The application of queuing theory extends well beyond waiting in line at a bank. It may take some creative thinking but if there is any sort of scenario where time passes before a particular event occurs, there is probably some way to develop it into a queuing model.

4.4 Queueing Applications

There are many applications of the theory of queues, most of which have been well documented in the literature of Probability, Operations Research, and Management Science. Some of the applications are (Banks, Supermarkets, machine repair, tool booths, Inventory control, the loading and unloading of ships, scheduling patients in hospital clinics, in computer fields etc.). Queueing theory is not only good for performance evaluation but can also use for security evaluation. Queueing theory has been used extensively in the banking industry to increase business by careful placement of merchandising materials while at the same time alleviating both the actual perceived amount of time a customer spends waiting in line. Queueing theory has been applied to computer simulation models to help with business decisions and problems. In a study by (Moss, G., 1987), queueing theory was used to assess the relationship among the number of pharmacy staff members, prescription dispensing process an outpatient waiting times. He provided the mathematical formula used in his research. Queueing theory is used in machine interference problems, machine service and repair model. We list papers with queueing applications. Analytical, numerical, different approximation and simulation techniques are used to obtain the particular characteristics.

4.5 Basic Structure of Queuing Models

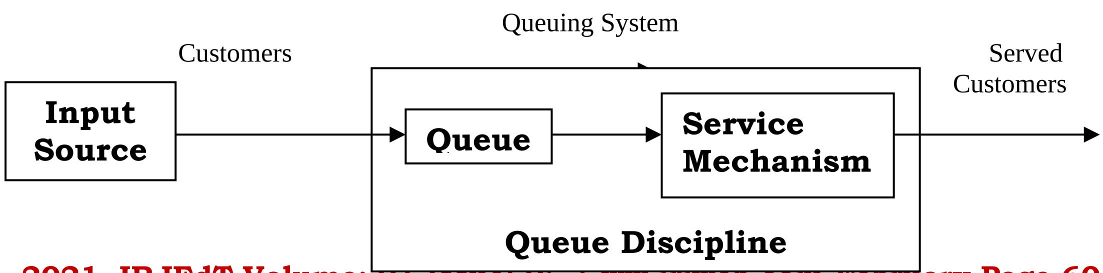

Figure 4.2: Basic Structure of Queuing Models

4.5.1 Input Source (calling population)

One characteristic of the input source is its size. The size is the total number of customers. The size may be infinite (default one) or infinite. When will each one (customer) arrived? Associate with a distribution usually, Poisson distribution (the number of customers generated until any specific time) or exponential distribution (inter-arrival time).

- A customer may be balking; who refuses to enter the system and is lost if the queues is too long.

4.5.2 Queue

- The queue is where customers wait before being served.
- A queue is characterized by the maximum permissible number of customers that it can contain. Queues may be infinite (default one) or infinite.

4.5.3 Queue Discipline:

- First-come-first-serve (FCFS) is normally used.
- Refers to the order in which members of the queues are selected for service.

4.5.4 Service Mechanism:

- Consists of one or more service facilities, each of which contains one or more parallel service channels, called servers.
- Service time usually defined by a probability distribution.
- At a given facility, the customer enters one of the parallel service channels and is served by that server.
- Most elementary model assumes one service facility with either one or finite number servers.

4.5 An Elementary Queuing Process

A single waiting line forms in the front of a single service facility, within which are stationed one or more servers. Each customer is served by one of the servers, perhaps after some waiting in the queue.

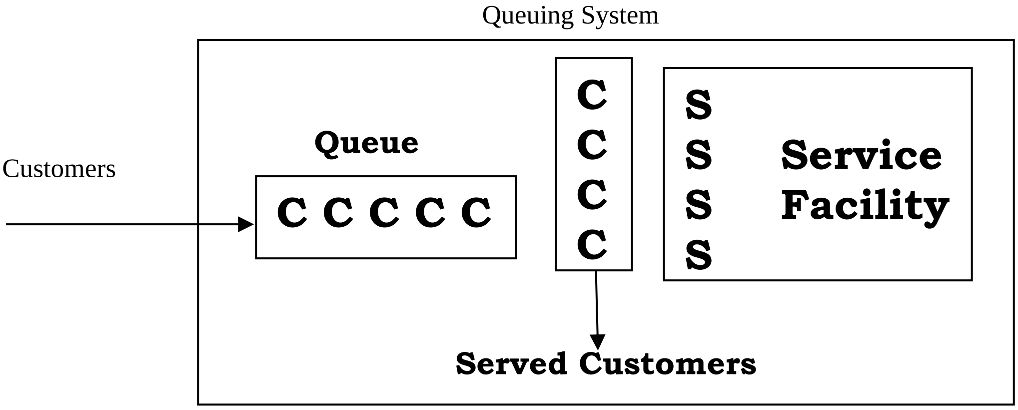

Figure 4.3: The prototype example of an elementary queuing process.

We usually label a quelling model as $\cdots/\cdots/\cdots$
Where:

- The first spot is for distribution of inter-arrival times.
- The second spot is for distribution of service time and
- The third spot is for number server.

Therefore, queuing model or systems are usually described by three values separated by slashes i.e.

$$Arrival\ Distribution/Service\ Distribution/Number\ of\ Servers$$

Where:
M =Markovian or exponentially distributed, which is the most widely used.
D =Deterministic or constant.
G =General or binomial distribution (Any arbitrary distribution allowed).
E_k= Erlang distribution.

The following are common queuing model:

1. $M/M/1$ (the simplest queuing model)

 Where:
 Arrival distribution $= M =$ Exponential distribution
 Service distribution $= M =$ Exponential distribution
 Number of servers= 1(One).

 Therefore, both the arrival time and service time of the above model $(M/M/1)$ are exponentially distributed with one server.

2. $M/D/1$:
 Where:
 Arrival distribution $= M =$ Exponential distribution
 Service distribution $= D =$ Deterministic
 Number of server= 1.

 Therefore, $M/D/1$ model has exponentially arrival times but fixed service time with one server.

3. $M/M/n$:
 Where:
 Arrival distribution $= M =$ Exponential distribution
 Service distribution $= M =$ Exponential distribution
 Number of services $= n =$ Multiple servers

Therefore, both the arrival time and service time of the above model $(M/M/N)$ are exponentially distributed with servers.

4.6 Features of a Single Queue Model

The single queue to single service point is otherwise known as a simple queue or single channel single phase. A single queue has the following features which are:

1. The arrival pattern is poisson
2. The service pattern is negative exponential
3. There is only one service channel
4. The traffic intensity (ρ) $is\ less\ than$ 1 (i.e. $\rho < 1$).
5. The queue discipline is First In First Out (FIFO)

The model deals with a queuing having a single server, here the arrivals are assumed to follow poisson distribution and the server time follows an exponential distribution. That is, the departure follows a poisson distribution. The system capacity has no limits and the customers are served on "FIFO" discipline i.e. First In First Out.
The following assumptions are made:

1. The average arrival rate of customers is λ
2. The average service rate of server is µ
3. The probability that there is no customers in the queue
4. The probability that there is at least one customer in the system.

4.7 Queuing Theory Equations

Definitions:
$\lambda =$ Arrival rate
$\mu =$ Service rate
$\rho =$ Traffic intensity
C = Number of service channels

M = Random Arrival / Service rate (Poisson)
D = Deterministic service rate (Constant rate).

Case 1:
$M/D/1$ case (Random Arrival, Deterministic Service, and one Service Channel). In some systems, the service time is always a constant. The $M/D/1$ model is used for constant service time. There is less randomness in the system and the wait time will be less. Some of the $M/D/1$ parameters are:

Expected or average queue length,

$$E(L_q) = \frac{(2\rho-\rho^2)}{2(1-\rho)} = \frac{\rho(2-\rho)}{2(1-\rho)} \tag{1}$$

Expected or average total time,

$$E(V) = T_q = \frac{(2-\rho)}{2\mu(1-\rho)} \tag{2}$$

Expected or average waiting time,

$$E(W) = T_w = \frac{\rho}{2\mu(1-\rho)} \tag{3}$$

Number in the system,

$$\left.\begin{matrix} Q = \frac{\rho^2}{2(1-\rho)} + \rho \\ or \\ Q = \lambda \times T_q \end{matrix}\right\} \tag{4}$$

Number waiting i.e. the average number of customers waiting,

$$\left.\begin{matrix} W = \frac{\rho^2}{2(1-\rho)} \\ or \\ W = \lambda \times T_w \end{matrix}\right\} \tag{5}$$

Case 2:
$\boldsymbol{M/M/1}$ Case (Random Arrival, Random Service and one Service channel).
The following queuing parameters are used to describe the model $M/M/$:

The average time in the system where s is the service time $(\boldsymbol{T_s})$,

$$\left.\begin{matrix} T_s = \frac{S}{1-\rho} \\ \boldsymbol{or} \\ T_s = \frac{1}{\mu-\lambda} \end{matrix}\right\} ;\, where\ \boldsymbol{S}\ is\ \text{the service time.} \tag{6}$$

The average time waiting i.e. the average time each customer wait in the queue (T_w),

$$\left.\begin{matrix} T_w = \frac{S\rho}{1-\rho} \\ \boldsymbol{or} \\ T_q = \frac{\lambda}{\mu(\mu-\lambda)} \end{matrix}\right\} \tag{7}$$

The number in the system N_s: This is the number of items (customers) in the queue and receiving service $(\boldsymbol{Q})$,

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$$\left.\begin{array}{l} N_s = \frac{\rho}{1-\rho} \\ \quad \textbf{or} \\ N_s = \frac{\lambda}{\mu-\lambda} \end{array}\right\} \tag{8}$$

The number waiting i.e. the average number of customers waiting (W),

$$\left.\begin{array}{l} N_q = \frac{\rho^2}{1-\rho} \\ \quad \textbf{or} \\ N_q = W = \frac{\lambda^2}{\mu(\mu-\lambda)} \end{array}\right\} \tag{9}$$

Remarks:

- Utilization (traffic intensity) can also be calculated from the arrival rate and the service time i.e.
$$\rho = \lambda \times S$$
Where: λ = Arrival rate and S = Service time
- It is important that the units of both the arrival rate and the service time be identical. It may be necessary to convert these values to common units.

Expected average total time:

$$E(V) = \frac{\rho}{\lambda(1-\rho)} \tag{10}$$

The expected number to be served:

$$E(n) = N_s - N_q \tag{11}$$

Case 3: (Multi-Channel Queuing System):

$M/M/C$ Model (Random Arrival, Random Service and C service channel rather than one server). This is a reasonable mode for

- A bank queue with multiple tellers
- A shared memory multiprocessor
- A filling station with multiple service points etc.

This is a system where arrangement is made to solve queue problem under emergency situation.

In this case, we will analyze the model with exponential inter – arrival times with mean $\frac{1}{\lambda}$, exponential service times with mean $\frac{1}{\mu}$ and C parallel identical servers. Customers are served in order of arrival. The occupation is rate per server. Formulas Relating to Multi-Channel System are as follows:

Traffic Intensity,

$$(\rho) = \frac{\lambda}{C\mu} \tag{12}$$

Where:

$\boldsymbol{C}$ is the number of Channels

$\boldsymbol{\lambda}$ is the arrival rate and

$\boldsymbol{\mu}$ is the service rate

The probability of queuing on the arrival,

$$P_{QOA} = \frac{(\rho C)^2}{C!(1-\rho)} \times P_o \tag{13}$$

The probability of not queuing on arrival,

$$P'_{QOA} = 1 - \frac{(\rho C)}{C!(1-\rho)} \times P_o \tag{14}$$

The probability of having zero customer in the system,

$$P_o = \left[\sum_{N=0}^{C-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!} \frac{1}{(1-\rho)}\right]^{-1} \tag{15}$$

The average expected queue length $E(Lq)$,

$$E(Lq) = \pi_w \frac{\rho}{(1-\rho)} \tag{16}$$

Where:

$$\pi_w = \frac{(c\rho)^c}{c!}\left[\sum_{N=0}^{C-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!} \frac{1}{(1-\rho)}\right]^{-1}$$

The expected or average number of customers in the system,

$$E(n) = N_s = \frac{(\rho C)^C}{C!(1-\rho)^2} \times P_\circ + (c\rho) \equiv E(Lq) + (\rho C) \tag{17}$$

The expected total time $E(v)$,

$$E(N) = T_t = \frac{E(n)}{\lambda} = \frac{N_s}{\lambda} \tag{18}$$

The waiting time $E(w)$,

$$E(w) = \pi_w \frac{1}{(1-\rho)} \frac{1}{c\mu} \tag{19}$$

The expected number of customer in the queue $E(Q)$,

$$E(Q) = E(w) = \frac{\rho(C\rho)^C}{C!(1-\rho)^2} P_\circ \tag{20}$$

5.0 Data Analysis

Total Service:

1st service time $= \mu_1 = 134$
2nd service time $= \mu_2 = 79$
3rd service time $= \mu_3 = 108$
∴Total service $= \mu = \mu_1 + \mu_2 + \mu_3 = 134 + 79 + 108 = 321$
∴Total service $= \mu^* = 321$.

Total Arrival

1st arrival time $= \lambda_1 = 243.53$
2nd arrival time $= \lambda_2 = 201.82$
3rd arrival time $= \lambda_3 = 246.55$
Total time arrival $= \lambda^* = \lambda_1 + \lambda_2 + \lambda_3 = 243.53 + 201.82 + 246.55 = 691.90$
∴Total time arrival $= \lambda^* = 691.90$

5.1 Computations on Single Queuing Model for individual Machine

For machine 1: The arrival rate (ar1):

Arrival rate $\lambda = \frac{Total\ time\ arrival\ for\ machine\ 1}{P_1} = \frac{\lambda_1}{P_1}$

```
ar = 243.53
p1 = 30
ar1 <- ar/p1
ar1

## [1] 8.117667
```

Service rate (Sr1):

$$\mu = \frac{1}{c\left(\frac{\mu_1}{P_1}\right)}$$

To compute for service rate (sr1):

```
t = 134
C = 3
ts <- t/p1
sr1 <- 1/(ts*C)
sr1

## [1] 0.07462687
```

To compute Traffic Intensity (rho1):

$$\rho = \frac{\lambda}{\mu}$$

```
rho1 <- ar1*sr1
rho1

## [1] 0.605796
```

$$0.6057\ minute \equiv 0.6057 \times 60 = 36.342\ seconds \cong 36\ seconds.$$

The average time in the system where S is the service time (T_s):

$$T_{S_1} = \frac{S_1}{1-\rho_1};\ \text{Where S is the service time}$$

The average time in the system (ts1):

```
ts1 <- sr1/(1-rho1)
ts1

## [1] 0.1893103
```

$$0.1893\ \text{minute} \equiv 0.1893 \times 60 = 11.358\ \text{seconds} \cong 11\ seconds.$$

The average time waiting i.e. the average time each customer wait in the queue (T_w):

$$T_{W_1} = \frac{S\rho_1}{1-\rho_1}$$

The average time customers spent on queue (Tw1):

```
Tw1 <- (sr1*rho1)/(1-rho1)
Tw1

## [1] 0.1146834
```

$$0.1147 \;\; minute \equiv 0.1147 \;\; \times \; 60 = \; 6.88 \, seconds \cong 7 \, seconds.$$

The number waiting i.e. the average number of customers waiting (W):

$$N_{q_1} = \frac{{\rho_1}^2}{1-\rho_1}$$

The average length of the queue(Nq1):

```
Nq1 <- (rho1^2)/(1-rho1)
Nq1

## [1] 0.9309617
```

The number of customer in the system $N_{S:}$ **This is the number of items (customers) in the queue and receiving service (Q):**

$$N_{s_1} = \frac{\rho_1}{1-\rho_1}$$

The average number of customers in the system (Ns1):

```
q <- 1-rho1
qNs1 <- rho1/1-rho1
Ns1 <- rho1/q
Ns1

## [1] 1.536758
```

Expected average total time:

$$E(v_1) = \frac{\rho_1}{\lambda(1-\rho_1)}$$

The expected average total time (Ev1):

```
Ev1 <- (rho1/(ar1*(1-rho1)))
Ev1

## [1] 0.1893103
```

$$0.1893 \;\; minute \equiv 0.1893 \;\times\; 60 = \; 11.35 \, seconds \cong 11 \, seconds.$$

The expected number to be served:

$$E(n_1) \; = \; N_{s_1} - N_{q_1}$$

The expected number to be served E(n1):

```
En1 <- Ns1-Nq1
En1

## [1] 0.605796
```

For machine 2:

Arrival rate (Ar2):

$$\lambda = \frac{Total\ time\ arrival\ for\ machine\ 2}{P_2} = \frac{\lambda_2}{P_2}$$

To compute the Arrival Rate (ar2) for Machine 2:

```
ta2=201.82
p2=25
ar2 <- ta2/p2
ar2

## [1] 8.0728
```

Service rate (Sr2):

$$\mu = \frac{1}{c\left(\frac{\mu_2}{P_2}\right)}$$

```
ts2=79
sr11 <- ts2/p2
sr2 <- 1/(sr11*C)
sr2

## [1] 0.1054852
```

Traffic intensity (Rho2):

$$\rho = \frac{\lambda}{\mu}$$

Traffic Intensity for machine 2 (rho2):

```
rho2 <- ar2*sr2
rho2

## [1] 0.8515612
```

$$0.8517\ minute \equiv 0.8517 \times\ 60 =\ 51.10\ seconds \cong 51\ seconds.$$

The average time in the system where s is the service time (T_s):

$T_{s_2} = \frac{S_2}{1-\rho_2}$ Where S is the service time

Average time in the system (Ts2):

```
Ts2 <- sr2/(1-rho2)
Ts2

## [1] 0.710631
```

$$0.7106\ minute \equiv\ 0.7106 \times\ 60 =\ 42.636\ seconds \cong 43\ seconds.$$

The average time waiting i.e. the average time each customer wait in the queue(T_w):

$$T_{W_2} = \frac{S\rho_2}{1-\rho_2}$$

Average time each customer waits in the queue (Tw2):

```
Tw2 <- (sr2*rho2)/(1-rho2)
Tw2

## [1] 0.6051458
```

$$0.6051 minute \equiv 0.6051 \times\ 60 =\ 36.306\ seconds \cong 36\ seconds.$$

The number waiting i.e. the average number of customers waiting (W):

$$N_{q_2} = \frac{{\rho_2}^2}{1-\rho_2}$$

The Average length of the queue (Nq2):

```
Nq2 <- rho2^2/(1-rho2)
Nq2

## [1] 4.885221
```

The number of customer in the system $N_{S:}$. This is the number of items (customers) in the queue and receiving service (Q):

$$N_{s_2} = \frac{\rho_2}{1-\rho_2}$$

```
Ns2 <- rho2/(1-rho2)
Ns2

## [1] 5.736782
```

Expected average total time:

$$E(v_2) = \frac{\rho_2}{\lambda(1-\rho_2)}$$

The expected average total time E(v2):

```
Ev2 <- (rho2/(ar2*(1-rho2)))
Ev2

## [1] 0.710631
```

$$0.7106\ minute \equiv 0.7106 \times\ 60\ =\ 42.636\ seconds \cong 42\ seconds.$$

The expected number to be served:

$$E(n_2)\ =\ N_{s_2} - N_{q_2}$$

The expected number to be served E(n2):

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```
En2 <- Ns2-Nq2
En2

## [1] 0.8515612
```

For machine 3:

Arrival rate (ar3):

$$\text{Arrival rate } \lambda = \frac{Total\ time\ arrival\ for\ machine\ 3}{P_3} = \frac{\lambda_3}{P_3}$$

```
ta3=246.55
p3=30
ar3 <- ta3/p3
ar3

## [1] 8.218333
```

Service rate (Sr3):

$$\text{Service rate } \mu = \frac{1}{c\left(\frac{\mu_3}{P_3}\right)}$$

```
s3=108
sr3 <- 1/(C*(s3/p3))
sr3

## [1] 0.09259259
```

Traffic Intensity (rho3):

$$\rho = \frac{\lambda}{\mu}$$

```
rho3 <- ar3*sr3
rho3

## [1] 0.7609568
```

$$0.7610\ minute \equiv 0.7610 \times 60 = 45.66\ seconds \cong 46\ seconds.$$

The average time in the system where s is the service time (T_s):

$$T_{s_3} = \frac{S_3}{1-\rho_3}\text{; Where S is the service time}$$

Average time in the system (Ts3):

```
Ts3 <- sr3/(1-rho3)
Ts3

## [1] 0.3873467
```

$$0.3873 minute \equiv 0.3873 \times 60 = 23.238\ seconds \cong 23\ seconds.$$

The average time waiting i.e. the average time each customer wait in the queue(T_w):

$$T_{W_3} = \frac{S\rho_3}{1-\rho_3}$$

Average time waiting (Tw3):

```
Tw3 <- (sr3*rho3)/(1-rho3)
Tw3

## [1] 0.2947541
```

$$0.2948\ minute \equiv 0.2948 \times 60 = 17.688\ seconds \cong 18\ seconds.$$

The number waiting i.e. the average number of customers waiting (W):

$$N_{q_3} = \frac{{\rho_3}^2}{1-\rho_3}$$

The average length of the queue(Nq3):

```
Nq3 <- rho3^2/(1-rho3)
Nq3

## [1] 2.422387
```

The number of customer in the system $N_{S:}$ This is the number of items (customers) in the queue and receiving service (Q):

$$N_{s_3} = \frac{\rho_3}{1-\rho_3}$$

Average Number of Customers in the System (Ns3):

```
Ns3 <- rho3/(1-rho3)
Ns3

## [1] 3.183344
```

Expected average total time $E(v_3)$:

$$E(v_3) = \frac{\rho_3}{\lambda(1-\rho_3)}$$

```
Ev3 <- (rho3/(ar3*(1-rho3)))
Ev3

## [1] 0.3873467
```

$$0.3873\ minute \equiv 0.3873 \times 60 = 23.238\ seconds \cong 23\ seconds.$$

The expected number to be served $E(n_3)$:

$$E(n_3) = N_{s_3} - N_{q_3}$$

The Expected Number of Customers to be Served E(n3):

```
En3 <- Ns3-Nq3
En3

## [1] 0.7609568
```

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Table 5.1: Summary of the Results for Simple Queuing System

Notation and Parameters	*Machine1*	*Machine2*	*Machine3*
Traffic intensity(ρ)	0.6058	0.85	0.7610
Arrival rate (λ)	8.12	8.0725	8.2183
Service rate(μ)	0.0746	0.1055	0.0926
Average time in the system (T_s)	11 *seconds*	42 *seconds*	23 *seconds*
Average length of the queue (Nq)	1 *customer*	5 *customers*	2 *customers*
Average time waiting (T_w)	7 *seconds*	36 *seconds*	18 *seconds*
Number of customers in the system (N_s)	2 *customers*	6 *customers*	3 *customers*
Expected average total time E(v)	11 *seconds*	42 *seconds*	23 *seconds*
Expected number to be served E(n)	1 *customer*	6 *customers*	1 *customer*

5.2 Computations for Multi-Channel Queuing System/Model

Service rate (Sr):

$$\mu = \frac{Total\ service}{P} = \frac{\mu^*}{P}$$

To Compute Service Rate (Sr) or (TSR):

```
TS=321
P=85
TSR <- C*(TS/P)
TSR

## [1] 11.32941
```

Arrival rate (Ar):

$$\lambda = \frac{Total\ time\ arrival}{P} = \frac{\lambda^*}{P}$$

To Compute Arrival Rate (AR):

```
TTR=691.90
TR <- TTR/P
TR

## [1] 8.14
```

Traffic intensity (RHO):

$$\rho = \frac{\lambda}{C\mu}$$

To Compute Traffic Intensity (RHO):

```
RHO <- TR/TSR
RHO

## [1] 0.7184839
```

$$0.7185 \text{ minute} \equiv 0.7185 \times 60 = 43.11 \text{ seconds} \cong 43 \textit{ seconds}.$$

The probability of having zero customer in the system:

$$P_o = \left[\sum_{N=0}^{C-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!} \frac{1}{(1-\rho)}\right]^{-1}$$

The Probability of Number Customer in the System(PO):

```
a <- (C*RHO)^0/factorial(0)
b <- (C*RHO)^1/factorial(1)
c <- (C*RHO)^2/factorial(2)
d <- sum(a,b,c)
e <- ((C*RHO)^C)/factorial(C)
f <- (1/(1-RHO))
PO <- (1/(d+(e*f)))
PO

## [1] 0.08766431
```

The expected queue length E(Lq):

$$\mathrm{E(Lq)} = \pi_w \frac{\rho}{(1-\rho)}$$

$$\text{Where: } \pi_w = \frac{(c\rho)^c}{c!} \left[\sum_{N=0}^{C-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!} \frac{1}{(1-\rho)}\right]^{-1}$$

```
L <- ((1/(((1-RHO)*e)+d))*e)
ELq <- L*(RHO/(1-RHO))
ELq

## [1] 1.326469
```

The expected or average number of customers in the system (NS):

$$\mathrm{E(n)} = \mathrm{N_s} = \frac{(\rho C)^C}{C!\,(1-\rho)^2} \times P_\circ + (c\rho)$$

```
N <- (RHO*(C*RHO)^C)
F <- (factorial(C)*(1-RHO)^2)
Ns <- ((N/F)*PO)+(C*RHO)
Ns

## [1] 3.481921
```

The expected total time E(v):

$$\mathrm{E(N)} = \mathrm{T_t} = \frac{\mathrm{E(n)}}{\lambda^*} = \frac{\mathrm{N_s}}{\lambda^*}$$

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The Expected Total Time (Tt):

```
Tt <- Ns/TR
Tt

## [1] 0.4277544
```

$$0.4278 \text{ minute} \equiv 0.4278 \times 60 = 25.668 \text{ seconds} \cong 26\ seconds.$$

The waiting time E(w):

$$\mathrm{E(w)} = \pi_{\mathrm{w}} \frac{1}{(1-\rho)} \frac{1}{c\mu}$$

```
EW  <- (L*f)*(1/TSR)
EW

## [1] 0.1629569
```

$$0.1630 \text{ minute} \equiv 0.1630 \times 60 = 9.78 \text{ seconds} \cong 10\ seconds.$$

The expected number of customer in the queue E(Q):

$$\mathrm{E(Q)} = \mathrm{E(w)} = \frac{\rho(C\rho)^{C}}{C!\,(1-\rho)^{2}} P_{\circ}$$

The Expected Number of Items in the Queue (NQ):

```
NQ <- ((N/F)*PO)
NQ

## [1] 1.326469
```

The probability of Queuing on the Arrival (QOA):

$$P_{QOA} = \frac{(\rho c)^{c}}{C!(1-\rho)} \times P_{o}$$

```
QOA=(((RHO*C)^C)/(factorial(C)*(1-RHO)))*PO
QOA

## [1] 0.5197365
```

The Probability of not Queuing on Arrival (NQOA):

$$P'_{QOA} = 1 - \left(\frac{(\rho c)^{c}}{C!(1-\rho)}\right) P_{0}$$

```
NQOA=1-QOA
NQOA

## [1] 0.4802635
```

Table 5.2: Summary of the Results for Multi-Channel Queuing System Model

Parameters	Estimates
Arrival Rate for channel 1: Machine 1 (λ_1)	243.53

Arrival Rate for Channel 2: Machine 2 (λ_2)	201.82
Arrival Rate for Channel 3: Machine 3 (λ_3)	245.55
First Channel Service Rate (μ_1)	134
Second Channel Service Rate (μ_2)	79
Third Channel Service Rate (μ_3)	108
Traffic intensity (rho) (ρ)	0.7178
The Probability of Queuing On the Arrival (P_{QOA})	0.52
The Probability of not Queuing on Arrival (P'_{QOA})	0.48
The Probability of Zero Customer in the System	0.088
The Average (Expected) Queue Length	1 *customers*
The Average Number Of Customers In The System (N_s)	3 *customers*
The Expected Total Time (T_t)	26 *seconds*
The Expected Waiting Time	10 *seconds*
The Expected Number of Customer in the Queue E(Q)	1 *customer*

6.0 Chi Square Goodness of Fit test for Arrival and Service Rate

The data collected was tested to show if it follows a Poisson and exponential distributions of arrival and service rate using chi square goodness of fit.

6.1 Chi Square Goodness of Fit test for Arrival Rate

Hypotheses for Server 1:

H_o: Arrival rate for the first server at GB Supermarket follows Poisson Distribution

H_A: Arrival rate for the first server at GB Supermarket does not follows Poisson Distribution

```
dat_server1.Arriv_Time <- read.csv('arrival.time.server1.csv')

test_out_server1 <- chisq.test(dat_server1.Arriv_Time$freq.1)
test_out_server1

##
##  Chi-squared test for given probabilities
##
## data:  dat_server1.Arriv_Time$freq.1
## X-squared = 1.6, df = 5, p-value = 0.9012
```

Decision Rule:

Reject H_o if $p - value \leq \alpha$. Otherwise do not reject it.

Conclusion:

Since $p - value = 0.9012 > 0.05 = \alpha$, then we have no reason to reject H_o and conclude that arrival rate for the first server at GB Supermarket really follows Poisson Distribution at 5% level of significance.

Hypotheses for Server 2:

H_o: Arrival rate for the second server at GB Supermarket follows Poisson Distribution

H_A: Arrival rate for the second server at GB Supermarket does not follows Poisson Distribution

```
dat_server2.Arriv_Time <- read.csv('arrival.time.server2.csv')
test_out_server2 <- chisq.test(dat_server2.Arriv_Time$freq.2)
test_out_server2

##
##  Chi-squared test for given probabilities
##
## data:  dat_server2.Arriv_Time$freq.2
## X-squared = 0.4, df = 4, p-value = 0.9825
```

Decision Rule:

Reject H_o if $p - value \leq \alpha$. Otherwise do not reject it.

Conclusion:

Since $p - value = 0.9825 > 0.05 = \alpha$, then we do not reject H_o and conclude that arrival rate for the second server at GB Supermarket really follows Poisson Distribution at 5% level of significance.

Hypotheses for Server 3:

H_o: Arrival rate for the third server at GB Supermarket follows Poisson Distribution

H_A: Arrival rate for the third server at GB Supermarket does not follows Poisson Distribution

```
dat_server3.Arriv_Time <- read.csv('arrival.time.server3.csv')
test_out_server3 <- chisq.test(dat_server3.Arriv_Time$freq.3)
test_out_server3

##
##  Chi-squared test for given probabilities
##
## data:  dat_server3.Arriv_Time$freq.3
## X-squared = 9.2, df = 5, p-value = 0.1013
```

Decision Rule:

Reject H_o if $p - value \leq \alpha$. Otherwise do not reject it.

Conclusion:

Since $p - value = 0.1013 > 0.05 = \alpha$, then we do not reject H_o and conclude that arrival rate for the third server at GB Supermarket really follows Poisson Distribution at 5% level of significance.

6.2 Chi Square Goodness of Fit test for Service Rate

Hypotheses for Server 1:

H_o: Service rate for the first server at GB Supermarket follows Exponential Distribution

H_A: Service rate for the first server at GB Supermarket does not follows Exponential Distribution

```
dat_server1.ST <- read.csv('server1.service.time.csv')
test_out_server1 <- chisq.test(dat_server1.ST$freq.1)
test_out_server1
```

```
##
##  Chi-squared test for given probabilities
##
## data:  dat_server1.ST$freq.1
## X-squared = 8, df = 5, p-value = 0.1562
```

Decision Rule:
Reject H_o if $p - value \leq \alpha$. Otherwise do not reject it.

Conclusion:
Since $p - value = 0.1562 > 0.05 = \alpha$, then we do not reject H_o and conclude that service rate for the first server at GB Supermarket follows Exponential Distribution at 5% level of significance.

Hypotheses for Server 2:
H_o: Service rate for the second server at GB Supermarket follows Exponential Distribution
H_A: Service rate for the second server at GB Supermarket does not follows Exponential Distribution

```
dat_server2.ST <- read.csv('server2.service.time.csv')
test_out_server2 <- chisq.test(dat_server2.ST$freq.2)
test_out_server2

##
##  Chi-squared test for given probabilities
##
## data:  dat_server2.ST$freq.2
## X-squared = 3.08, df = 5, p-value = 0.6877
```

Decision Rule:
Reject H_o if $p - value \leq \alpha$. Otherwise do not reject it.

Conclusion:
Since $p - value = 0.6877 > 0.05 = \alpha$, then we do not reject H_o and conclude that service rate for the second server at GB Supermarket follows Exponential Distribution at 5% level of significance.

Hypotheses for Server 3:
H_o: Service rate for the third server at GB Supermarket follows Exponential Distribution
H_A: Service rate for the third server at GB Supermarket does not follows Exponential Distribution

```
dat_server3.ST <- read.csv('server3.service.time.csv')
test_out_server3 <- chisq.test(dat_server3.ST$freq.3)
test_out_server3

##
##  Chi-squared test for given probabilities
##
## data:  dat_server3.ST$freq.3
## X-squared = 10.133, df = 6, p-value = 0.1191
```

Decision Rule:
Reject H_o if $p - value \leq \alpha$. Otherwise do not reject it.

Conclusion:
Since $p - value = 0.1191 > 0.05 = \alpha$, then we do not reject H_o and conclude that service rate for the third server at GB Supermarket follows Exponential Distribution at 5% level of significance.

7.0 Results and Discussions

The results of the analysis on the data set for Single Queuing System revealed that the traffic intensity of each of three machines/service points were $0.6058, 0.8516$ *and* 0.7610 respectively. The average time waiting on the queue for the first three machines/service points were 7 seconds, 36 seconds and 18 seconds respectively. That is, the first server/service point is more efficient than the others. The service rate for machine 1, machine 2 and machine 3 were $0.0746, 0.1055$ *and* 0.0926 respectively; which implies that the first and third servers are faster in attending to their customers. The expected number to be served by machine 1, machine 2 and machine 3 were 1 customer, 6 customers and 1 customer respectively. The expected average time of each of the three service points in the system were 11 seconds, 42 seconds and 23 seconds respectively. Furthermore, the average rate of arrival and average rate of the service for the multiple channel were 8.14 and 11.34 respectively. The traffic intensity, that is, the rate at which the servers at Grocery Bazaar supermarket is busy was 0.7178, this implies that the traffic intensity is relatively high. The probability of queuing on arrival and not queuing on arrival were 0.52 *and* 0.48 respectively. Therefore, the probability of queuing on arrival of the customers and not queuing on arrival was high. The probability of zero customer in the system was 0.088, Average number of customer in the queue was 1, Average number of customers in the system and Average time each customer spent on queue were 3 customers and 10 *seconds* respectively. The Chi-square goodness of fit test results revealed that the arrival and service rate at GB supermarket really followed the Poisson and Exponential distribution respectively at 5% level of significance.

8.0 Conclusion

Looking at the data analysis we can see clearly that the traffic intensity for the combine service points (multiple channel) is relatively high which implies how the servers are busy i.e. there is queue in Grocery Bazaar supermarket but the service render is low. However, from the analysis, despite the long queue and few services point i.e. 3 channels we discovered that customers spent more time in the service due to unavailable of enough service channel, delay in looking for change by the attendants and insecurities of conducting the queue and customers that refuse to leave the queue, after service had been rendered. Delay is also found in the used of POS (Point of Sales) machine by bad network and programming of price tag of not corresponding to the one program in the system, with this the scanner is unavailable to scan, which causes delay most especially in Grocery Bazaar supermarket. Hence, from the analysis, the first service point is faster, follow by the third service point and lastly the second service point in serving the customer.

9.0 Recommendations

Technologically, the equipment innovation used should be supply more to further greatly improved so as to really make sure the service rate gets recruitment of employees to render more service because the same person that sells also operate computer for scanning the varieties and also collect money which this leads to the long queue. Adopting this, it will reduce or minimal the delay in the queuing. The management should increase the service channel, performance, reliability in terms of service rendered and added feature i.e. giving their customers souvenir during festival time and guiding of the customers by introducing the First-In-First-Out (FIFO) queuing discipline. There must also be a thorough check up for the attendant by the management in the Grocery Bazaar (GB) supermarket. By putting all these into consideration, there will be effectiveness and efficiency in the Grocery Bazaar (GB) supermarket.

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